Continental-Scale, High-Order, High Spatial-Resolution Ice Flow Modeling Based on Graphics Processing Units (GPUs)

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Global mean sea level is rising at an unprecedented rate, posing a significant threat to coastal communities and ecosystems worldwide. The ice discharge from the Antarctic Ice Sheet (AIS) contributes significantly to rising sea levels; however, its dynamic response to climate change remains a fundamental uncertainty [1] . Ice-sheet models that accurately capture dynamic response are essential tools for sea-level rise projections. Traditionally, these models are based on partial differential equations representing mass, momentum, energy conservation, and the Glen-Nye flow law [2]. There are seven unknown parameters in these models: the three ice velocity components (v_x, v_y, v_z), ice pressure (p), ice temperature (T), and surface and bed elevations (s, b). The ice sheet model is initialized with geometry, velocity, and temperature fields. Temperature is computed using the energy conservation equation, followed by updating the ice viscosity. The ice velocity is then computed using the momentum conservation equation. An additional step in prognostic models is to apply the mass-conservation equation to calculate ice geometry for the next time increment. Other ice sheet model capabilities include capturing subglacial hydrology, calving, and glacial isostatic adjustment [3]–[7].

Quantifying uncertainties in future Antarctic ice-sheet evolution requires running high-fidelity simulation ensembles; however, conventional central processing units (CPUs) limit the time available. Aside from subglacial hydrology, solving time-independent momentum-balance equations to predict ice velocity is the most computationally expensive part of ice-sheet simulations in terms of memory and execution time [8] .

The ice velocity component of the ice sheet models can be classified into four categories: (1) 2-D Shallow Ice Approximation [9], (2) MacAyeal-Morland's 2-D Shallow-Shelf Approximation (SSA) [10], [11], (3) Blatter-Pattyn's 3-D Higher-Order (HO) [12], [13], and (4) 3-D Full Stokes [14]. All four categories share the same set of partial differential equations but employ different assumptions that simplify them to varying degrees. The 2-D models, shallow ice and shallow shelf, solve for ice velocity (v_x, v_y). These models assume that ice velocity is depth-independent and are best suited when vertical variations in ice velocity are negligible. The 3-D higher-order models include vertical variations in ice velocity and the longitudinal and lateral forces on the ice sheet; however, they decorrelate the horizontal velocity components from the vertical component. This model solves for only two unknowns (v_x, v_y) as a result; however, it represents flow more accurately over most of the sheet, except near the grounding lines, where these assumptions are not strictly valid. The full Stokes model provides a more accurate representation of the grounding lines because it includes a complete description of mechanical ice-sheet deformation, solving for ice velocity (v_x, v_y, v_z) and ice pressure (p); however, it requires a significant amount of computational resources in terms of computer memory and execution time at large scales.

Several studies have employed lower-fidelity models, i.e., momentum-balance approximations such as 2-D SIA or 2-D SSA, or spatial resolutions of 1 km or greater near grounding lines, to keep computational resources manageable when running simulation ensembles of continental-scale Antarctica forward in time [15] . These studies partially quantify the uncertainties in future Antarctic ice sheet evolution. More recently, machine learning techniques have been applied to develop surrogate models and to mitigate the high computational costs of ice-velocity prediction [8] [16] [17] [18] [19] . These techniques show promise; however, they have not yet been validated at a continental scale.

In this study, we develop numerical algorithms that leverage the parallel processing capabilities of graphics

processing units (GPUs) and implement them within the Ice-sheet and Sea-level System Model (ISSM) 3-D HO stress balance module. Our configurations are based on the ISSM Antarctic model used in the Ice Sheet Model Intercomparison for CMIP6 (ISMIP6), featuring approximately 10^7 degrees of freedom (DoFs), as well as a higher-resolution setup with approximately 10^8 DoFs [15]. We compare the GPU-accelerated implementation of the 3-D HO ISSM stress balance module to the standard CPU-based implementation. The performance metrics evaluated include predictive accuracy, computational efficiency, and robustness. Our GPU implementation produces results consistent with the standard CPU implementation while substantially reducing computational expense, achieving speedups of 3x and 8x for the two Antarctic Ice Sheet configurations with 10^7 and 10^8 DoFs, respectively. Furthermore, we observed GPU-CPU consistency across a range of stress regimes—including extreme rheological and basal sliding contrasts, nonlinear friction formulations, and geometrically complex settings such as grounding lines and ice shelves—under realistic Antarctic flow conditions.

The overarching goal is to enable high-fidelity ensembles of Antarctic Ice Sheet simulations to better quantify uncertainties in sea-level rise projections. These findings have not been available due to computational costs; however, they are urgently needed as the Antarctic Ice Sheet loses mass at an increasing rate and will significantly benefit process-oriented and sea-level-projection studies over the coming decades. The methods developed will enable the ice sheet community to quantify projection uncertainty with greater confidence, better identify its sources, and determine which satellite measurements are required to reduce it.

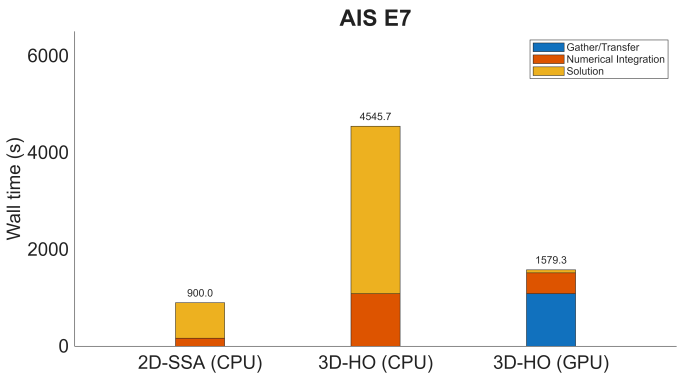


Fig. 1. Performance comparison of the GPU implementation with the standard CPU implementation employing wall time for the AIS with 10^7 DoFs. We obtain timings using NVIDIA A100 GPUs.

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